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USE OF AN AVERAGING METHOD IN COMPUTING A PLATE REINFORCED WITH A STRINGER*

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A three-dimensional problem of the theory of elasticity is considered for a half-layer reinforced with a stringer and end-loaded with a symmetric load. The averaging approach is used, together with the method of asymptotic integration. It is shown that the state of stress in the structure can be separated into an averaged state described by two-dimensional equations for a reinforced plate, and a supplementary state governed by three-dimensional effects within the region of the rib. The distribution of the contact stress across the stringer width is obtained, and the character of the stress state near the zone of contact studied. It is shown that the supplementary stresses are of the same order as the shear stress of the averaged state.

1. In solving the reinforced plates and shells, the influence of the ribs is accounted for, as a rule, by inserting additional terms into the equations of equilibrium, the terms containing the distribution functions of the contact stresses across the width of the region of contact /1/. However, when solving the practical problems, we either assume that the distribution is uniform, or that the rib and the casing interact with each other along a line. The approximation proposed relates to the fact that the contact stress distribution is not known and can only be determined by considering the rib and the casing with the framework of the three-dimensional theory of elasticity, which presents appreciable difficulties.

We consider an isotropic half-layer reinforced by a stringer (Fig.1). A load periodic in x_2 and symmetric about the x_3-axis, with zero moment about the neutral plane, is applied at the end $x_1 = 0$. The half-wave length πl of the load is much larger than the plate thickness $2h$ and the transverse dimensions $2b$ and d of the stringer. Taking into account the character of the load, we can assume that the only stresses appearing between the plate and the rib are longitudinal contact stresses $q_1(x_1, x_2)$ directed along the x_1-axis. Then the problem can be described by the following system of equations:

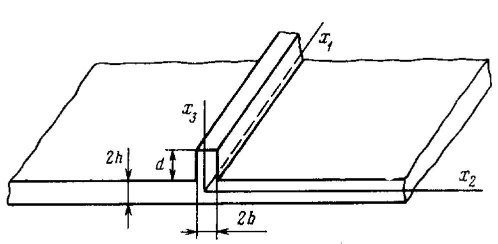

Fig.1

$$L_1(\mathbf{u}^n) + \delta(x_3 - h) H_1(x_2) q_1(x_1, x_2) = 0, \quad L_k(\mathbf{u}^n) = 0 \tag{1.1}$$

$$L_1(\mathbf{u}^p) - \delta(x_3 - h) q_1(x_1, x_2) = 0, \quad L_k(\mathbf{u}^p) = 0 \tag{1.2}$$

$$\mathbf{u}^n = \{u_1^n, u_2^n, u_3^n\}, \quad \mathbf{u}^p = \{u_1^p, u_2^p, u_3^p\}$$

$$L_i(\mathbf{u}) \equiv (\lambda + \mu) \frac{\partial}{\partial x_i} \left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} \right) + \mu \Delta u_i,$$

$$H_1(x_2) = H(x_2 + b) - H(x_2 - b)$$

Here $L_i(\mathbf{u})$ are the Lamé operators, $\mathbf{u}^n$ and $\mathbf{u}^p$ are the plate and rib displacement vectors, $\delta(x_3)$, $H(x_2)$ are the generalized Dirac and Heaviside functions, respectively, λ and μ are the elastic Lamé constants and the indices i, k, j assume, for now on, the values of 1,2,3; 2,3 and 1,2 respectively. The boundary conditions at the rib edges and the plate foundations are homogeneous.

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To solve the problem we employ the concept of averaging, and thus obtain the intiial approximation using the averaged relationships. To formulate the averaged problem for a plate we assume that $q_1(x_1, x_2)$ acts in a concentrated manner along the mean line of contact, that the stress $\sigma_{33}{}^n = 0$, and integrate (1.1) across the thickness. This, together with the conditions at the planes $x_3 = \pm h$, yields

$$L_{1*}(u_{1*}^n, u_{2*}^n) + \frac{\delta(x_2)}{2h}\bar{q}_1(x_1) = 0, \quad L_{2*}(u_{1*}^n, u_{2*}^n) = 0 \tag{1.3}$$

$$u_{j*}^n(x_1, x_2) = \frac{1}{2h}\int_{-h}^{h} u_j^n(x_1, x_2, x_3)\,dx_3, \quad \bar{q}_1(x_1) = \int_{-b}^{b} q_1(x_1, x_2)\,dx_2 \tag{1.4}$$

where L_{1*} and L_{2*} are the Lamé operators in the case of a generalized plane state of stress.

Assuming that $\sigma_{22}{}^p = \sigma_{33}{}^p = 0$, intergrating (1.2) over the area of transverse cross section of the stringer F, and taking into account the conditions at the side edges, we obtain the following averaged expression for the rib:

$$EF\frac{\partial^2 u_{1*}^p}{\partial x_1{}^2} - \bar{q}_1(x_1) = 0, \qquad u_{1*}^p(x_1) = \frac{1}{F}\iint_F u_1{}^p(x_1, x_2, x_3)\,dF \tag{1.5}$$

Assuming that the displacements of the plate and rib coincide along the line of contact $(u_{1*}{}^p(x_1) = u_{1*}{}^n(x_1, 0))$ we obtain, from (1.3) and (1.5),

$$L_{1*}(u_{1*}^n, u_{2*}^n) + \frac{EF}{2h}\delta(x_2)\frac{\partial^2 u_{1*}^n}{\partial x_1{}^2} = 0, \quad L_{2*}(u_{1*}^n, u_{2*}^n) = 0 \tag{1.6}$$

The boundary conditions for (1.6) are obtained by integrating, with respect to x_3, the boundary conditions specified at the end $x_1 = 0$. The problem formulated in this manner represents a problem of generalized plane stress state for a reinforced plate. From the conditions $\sigma_{33*}{}^n = 0$ and $\sigma_{22*}{}^p = \sigma_{33*}{}^p = 0$ we obtain (ν is the Poisson's ratio)

$$u_{3*}^n = -\varkappa x_3\psi, \quad u_{2*}^p = -\nu x_2\frac{\partial u_{1*}^p}{\partial x_1}, \quad u_{3*}^p = -\nu(x_3 + \varkappa h)\frac{\partial u_{1*}^p}{\partial x_1}$$

$$\psi = \frac{\partial u_{1*}^n}{\partial x_1} + \frac{\partial u_{2*}^n}{\partial x_2}, \quad \varkappa = \frac{\nu}{1-\nu}$$

and the displacements of the stringer and plate coincide on the line $x_2 = 0$, $x_3 = h$.

2. Let us adopt the values $u_{i*}{}^n$ as the initial approximation to the solution of the system (1.1), and $u_{1*}{}^p(x_1) = u_{1*}{}^n(x_1, 0)$, $u_{2*}{}^p$, $u_{3*}{}^p$ as the initial approximation to the solution of the system (1.2), and write the solutions sought in the form

$$\mathbf{u}^n = \mathbf{u}_*{}^n + \mathbf{V}, \quad \mathbf{u}^p = \mathbf{u}_*{}^p + \mathbf{W}; \quad \mathbf{V} = \{V_1, V_2, V_3\}, \quad \mathbf{W} = \{W_1, W_2, W_3\} \tag{2.1}$$

where $\mathbf{V}$ and $\mathbf{W}$ are the additional plate and rib displacement vectors respectively. Substituting (2.1) into (1.1), (1.2) and into the boundary conditions at the plate foundations and rib edge, we obtain

$$L_1(\mathbf{V}) = \varkappa\mu\frac{\partial\psi}{\partial x_1} - \delta(x_3 - h)H_1(x_2)q_1(x_1, x_2) + \frac{\delta(x_2)}{2h}\bar{q}_1(x_1) \tag{2.2}$$

$$L_2(\mathbf{V}) = \varkappa\mu\frac{\partial\psi}{\partial x_2}, \quad L_3(\mathbf{V}) = \varkappa\mu x_3\Delta_1\psi; \quad \Delta_1 = \frac{\partial^2}{\partial x_1{}^2} + \frac{\partial^2}{\partial x_2{}^2}$$

$$\sigma_{33}(\mathbf{V})|_{x_3=\pm h} = 0, \quad \sigma_{j3}(\mathbf{V})|_{x_3=\pm h} = \pm\varkappa h\frac{\partial\psi}{\partial x_j}$$

$$L_1(\mathbf{W}) = 2\mu\nu\frac{\partial^2 u_{1*}^p}{\partial x_1{}^2} - \frac{\bar{q}_1(x_1)}{F} + \delta(x_3 - h)q_1(x_1, x_2) \tag{2.3}$$

$$L_k(\mathbf{W}) = \mu\nu x_k\frac{\partial^3 u_{1*}^p}{\partial x_1{}^3}$$

$$\sigma_{12}(\mathbf{W})|_{x_2=\pm b} = \pm\nu\mu b\frac{\partial^2 u_{1*}^p}{\partial x_1{}^2}, \quad \sigma_{22}(\mathbf{W})|_{x_2=\pm b} = \sigma_{23}(\mathbf{W})|_{x_2=\pm b} = 0$$

$$\sigma_{13}(\mathbf{W})|_{x_3=h} = \nu\mu h\frac{\partial^2 u_{1*}^p}{\partial x_1{}^2}, \quad \sigma_{13}(\mathbf{W})|_{x_3=h+d} = \nu\mu(h+d)\frac{\partial^2 u_{1*}^p}{\partial x_1{}^2}$$

$$\sigma_{33}(\mathbf{W})|_{x_3=h,\,h+d} = \sigma_{23}(\mathbf{W})|_{x_3=h,\,h+d} = 0$$

The boundary value problems (2.2) and (2.3) formulated here appear not to be simpler than the initial problems. They have however the advantage that the variables $\mathbf{V}$ and $\mathbf{W}$ sought, as well as the corresponding stresses, can be written in the form of two terms each of which varies rapidly either with respect to the coordinates x_2 and x_3, or with respect to x_3 only (the first term for the plate). This enables us to apply, with success, the asymptotic methods to the study of the additional state of stress.

We seek the solution of the system (2.2) in the form of a sum of solutions of the systems (2.4) and (2.5):

$$L_j(\mathbf{V}^{(1)}) = \varkappa\mu \frac{\partial\psi}{\partial x_j}, \quad L_3(\mathbf{V}^{(1)}) = \varkappa\mu x_3 \Delta_1 \psi \tag{2.4}$$

$$L_1(\mathbf{V}^{(2)}) = \frac{\delta(x_2)}{2h}\bar{q}_1(x_1) - \delta(x_3 - h) H_1(x_2) q_1(x_1, x_2), \quad L_k(\mathbf{V}^{(2)}) = 0 \tag{2.5}$$

A particular solution of the system (2.4) satisfying the boundary conditions of (2.2) is obtained by the asymptotic method using the stretching of the coordinate x_3 /2/, and has the form

$$V_j^{(1)} = \frac{\varkappa}{2}\left(x_3^2 - \frac{h^2}{3}\right)\frac{\partial\psi}{\partial x_j}, \quad V_3^{(1)} = 0 \tag{2.6}$$

This solution satisfies the system (2.4) with the accuracy of up to the terms $\sim \nu^2\varepsilon_1$ compared with 1, and the boundary conditions of (2.2) with the accuracy of $\nu^2\varepsilon_1^2$ (ε_1 is a small parameter equal to h/l).

To find the solution of the system (2.5) satisfying the homogeneous boundary conditions at the foundations, we make use of the fact that the right-hand part of the system (2.5) can be regarded as a mass force which is self-equilibrated in the region $|x_3| \leqslant h$, $|x_2| \leqslant b$ for every x_1

$$\int_{-h}^{h}\int_{-b}^{b}\left[\frac{\delta(x_2)}{2h}\bar{q}_1(x_1) - \delta(x_3 - h) H_1(x_2) q_1(x_1, x_2)\right] dx_2\, dx_3 \equiv 0$$

This means that in accordance with the Saint-Venant principle, the corresponding stress state will be rapidly attenuated with increasing distance from the region of contact, i.e. it will vary rapidly in x_2 and x_3. Therefore we introduce the change of coordinates $x_2 = \varepsilon_1\bar{x}_2$, $x_3 = \varepsilon_1\bar{x}_3$ and write the unknown variables in the form

$$V_i^{(2)} = \sum_{n=0}^{\infty} V_{in}\varepsilon_1^n \tag{2.7}$$

Substituting these into the system (2.5) and the homogeneous boundary conditions at the foundations, and collecting together the terms accompanying like powers of ε_1, we obtain

$$\mu\left(\frac{\partial^2 V_{1n}}{\partial \bar{x}_2^2} + \frac{\partial^2 V_{1n}}{\partial \bar{x}_3^2}\right) = -(\lambda + \mu)\frac{\partial}{\partial x_1}\left(\frac{\partial V_{2n-1}}{\partial \bar{x}_2} + \frac{\partial V_{3n-1}}{\partial \bar{x}_3} + \frac{\partial V_{1n-2}}{\partial x_1}\right) - \tag{2.8}$$

$$\mu\frac{\partial^2 V_{1n-2}}{\partial x_1^2} + \begin{cases} \varepsilon_1^2\left[\dfrac{\delta(x_2)}{2h}\bar{q}_1 - \delta(x_3 - h) H_1(x_2) q_1\right], & n = 0 \\ 0, & n \neq 0 \end{cases}$$

$$\left.\frac{\partial V_{1n}}{\partial \bar{x}_3}\right|_{x_3 = \pm h} = \left.\frac{\partial V_{3n-1}}{\partial x_1}\right|_{x_3 = \pm h}$$

$$(\lambda + \mu)\frac{\partial}{\partial \bar{x}_k}\left(\frac{\partial V_{2n}}{\partial \bar{x}_2} + \frac{\partial V_{3n}}{\partial \bar{x}_3}\right) + \mu\left(\frac{\partial^2 V_{kn}}{\partial \bar{x}_2^2} + \frac{\partial^2 V_{kn}}{\partial \bar{x}_3^2}\right) = \tag{2.9}$$

$$-(\lambda + \mu)\frac{\partial^2 V_{1n-1}}{\partial x_1 \partial \bar{x}_k} - \mu\frac{\partial^2 V_{kn-2}}{\partial x_1^2}$$

$$\left.(1 - \nu)\frac{\partial V_{3n}}{\partial \bar{x}_3} + \nu\frac{\partial V_{2n}}{\partial \bar{x}_2}\right|_{x_3 = \pm h} = -\nu\left.\frac{\partial V_{1n-1}}{\partial x_1}\right|_{x_3 = \pm h},$$

$$\left.\frac{\partial V_{2n}}{\partial \bar{x}_3} + \frac{\partial V_{3n}}{\partial \bar{x}_2}\right|_{x_3 = \pm h} = 0$$

where $n = 0, 1, 2\ldots$; and V_{in} are zero when $n < 0$. We see that the three-dimensional problem has decomposed into a sequence of two-dimensional boundary value problems (2.8), (2.9) for the half-strip $|x_3| \leqslant h$, $|x_2| < \infty$.

In the zero approximation ($n = 0$) the boundary value problem (2.9) yields the trivial solution ($V_{20} = V_{30} = 0$), and (2.8) has the form

$$\frac{\partial^2 V_{10}}{\partial \bar{x}_2^2} + \frac{\partial^2 V_{10}}{\partial \bar{x}_3^2} = \frac{\varepsilon_1^2}{\mu}\left[\frac{\delta(x_2)}{2h}\bar{q}_1 - \delta(x_3 - h) H_1(x_2) q_1\right] \qquad (2.10)$$
$$\left.\frac{\partial V_{10}}{\partial \bar{x}_3}\right|_{x_3=\pm h} = 0, \quad V_{10}|_{x_2\to\pm\infty} \to 0$$

The solution of the boundary value problem (2.10) is obtained with the help of the integral Fourier transforms /3/ with infinite (in x_2) and finite (in x_3) limits, in the form (basic coordinates are used for convenience)

$$V_1^{(2)} \approx V_{10} = \frac{1}{2\pi\mu}\int_{-\infty}^{\infty}\left[\frac{\operatorname{ch} p\gamma x_3}{p \operatorname{sh} p\gamma\pi}\int_{-b}^{b} q_1(x_1, x_2)\cos px_2\, dx_2 - \frac{\bar{q}_1(x_1)}{2hp^2}\right]\cos px_2\, dp, \qquad \gamma = \frac{2h}{\pi} \qquad (2.11)$$

The subsequent approximations $V_i^{(2)}$ can be obtained without any fundamental difficulties. It should be noted that the solution (2.11) fails in the region $|x_3| \leqslant h$, $|x_2| \lesssim b$, $x_1 \lesssim 2h$ where the state of stress varies at the same rate in all three coordinates.

The additional solution for the rib, as well as for the plate, consists of a particular solution of the first type

$$W_1^{(1)} = \frac{\nu}{2}\left(x_2^2 + x_3^2 - \frac{b^2 + d^2}{3}\right)\frac{\partial^2 u_{1*}^p}{\partial x_1^2}, \quad W_k^{(1)} = 0 \qquad (2.12)$$

intended for the compensation of discrepancies in the boundary conditions in (2.3), and the solution of the system

$$L_1(\mathbf{W}^{(2)}) = -\frac{\bar{q}_1(x_1)}{F} + \delta(x_3 - h) q_1(x_1, x_2), \quad L_k(\mathbf{W}^{(2)}) = 0 \qquad (2.13)$$

with homogeneous boundary conditions at the side edges. To obtain the latter, we take into account the fact that for every x_1

$$\iint_F \left[\frac{\bar{q}_1(x_1)}{F} - \delta(x_3 - h) q_1(x_1, x_2)\right] dF \equiv 0$$

perform the change of coordinates $x_2 = \varepsilon_2 x_2^*$, $x_3 = \varepsilon_2 x_3^*$ and write the solution in the form

$$W_i^{(2)} = \sum_{m=0}^{\infty} W_{im}\varepsilon_2^{m} \qquad \left(\varepsilon_2 = \max\left\{\frac{b}{l}, \frac{d}{l}\right\}\right)$$

As a result, we obtain a recurrent process analogous to (2.8) and (2.9). The principal component $W^{(2)}$ is found from the following boundary value problem:

$$\frac{\partial^2 W_{10}}{\partial x_2^{*2}} + \frac{\partial^2 W_{10}}{\partial x_3^{*2}} = -\frac{\varepsilon_2^2}{\mu}\left[\frac{\bar{q}_1}{F} - \delta(x_3 - h) q_1\right]$$
$$\left.\frac{\partial W_{10}}{\partial x_2^*}\right|_{x_2^*=\pm b/\varepsilon_2} = 0, \quad \left.\frac{\partial W_{10}}{\partial x_3^*}\right|_{x_3^*=h/\varepsilon_2,\,(d+h)/\varepsilon_2} = 0$$

the solution of which is obtained using Fourier transform with finite limits /3/. The solution has the form

$$W_1^{(2)} \approx W_{10} = -\frac{d\bar{q}_1(x_1)}{b\mu}\left[\frac{(x_3 - h)^2}{4d^2} - \frac{(x_3 - h)}{2d} + \frac{1}{6}\right] - \qquad (2.14)$$
$$\frac{2}{\pi\mu}\sum_{t=1}^{\infty}\frac{\operatorname{ch}\alpha t\pi\left(\dfrac{x_3 - h}{d} - 1\right)}{t \operatorname{sh}\alpha t\pi}\int_{-b}^{b} q_1(x_1, x_2)\cos\frac{t\pi}{2b}(x_2 + b)\, \times$$
$$dx_2 \cos\frac{t\pi}{2b}(x_2 + b), \qquad \alpha = \frac{d}{2b}$$

It should be noted that the additional solutions (2.6), (2.11) and (2.12), (2.14) obtained contribute towards the boundary conditions at the end face only the self-equilibrating errors, and their validity in the integral sense is not therefore affected. The boundary conditions at the end face can be satisfied exactly by constructing a state of stress of the boundary layer type /2,4/.

3. The results obtained show that when a load of the type shown is applied, longitudinal displacements u_1^n and u_1^p prevail in the region of contact, and the transverse displacements are of higher order of smallness. For this reason we determine the unknown contact stress $q_1(x_1, x_2)$ by writing the conditions of equality of the longitudinal displacements of the half-layer and rib at every point of the area of contact, as

$$u_{1*}^n(x_1, x_2) + V_1^{(2)}(x_1, x_2, h) = u_{1*}^p(x_1) + W_1^{(2)}(x_1, x_2, h) \tag{3.1}$$

The relation (3.1) must hold for every x_1 and $|x_2| \leqslant b$ and the average displacement of the rib $u_{1*}^p(x_1) \equiv u_{1*}^n(x_1, 0)$ does not vary across the strip width. The displacements $V_1^{(1)}$ and $W_1^{(1)}$ in (3.1) are not taken into account, since they are of second order of smallness compared with the terms given. Substituting into (3.1) the expressions for $V_1^{(2)}$ (2.11) and $W_1^{(2)}$ (2.14), we obtain the following integral equation:

$$\int_{-\infty}^{\infty} \frac{\operatorname{cth} p\gamma\pi}{p} \int_{-b}^{b} q_1(x_1, x_2) \cos px_2\, dx_2 \cos px_2\, dp + \tag{3.2}$$

$$4 \sum_{t=1}^{\infty} \frac{\operatorname{cth} \alpha t\pi}{t} \int_{-b}^{b} q_1(x_1, x_2) \cos \frac{t\pi}{2b}(x_2 + b)\, dx_2 \cos \frac{t\pi}{2b}(x_2 + b) = \varphi(x_1, x_2) + \frac{\bar{q}_1(x_1)}{2h} \int_{-\infty}^{\infty} \frac{\cos px_2}{p^2}\, dp$$

$$\varphi(x_1, x_2) = 2\pi\mu\, [u_{1*}^n(x_1, x_2) - u_{1*}^n(x_1, 0)] + \frac{\bar{q}_1\pi d}{3b}, \qquad -b \leqslant x_2 \leqslant b$$

(x_1 appears in the equation as a parameter).

To solve (3.2) we write the unknown contact stress in the form

$$q_1(x_1, x_2) = a_0(x_1) + \sum_{m=1}^{\infty} a_m(x_1) \cos \frac{m\pi x_2}{b} \tag{3.3}$$

Here only the even harmonics are retained, since in this case the function $q_1(x_1, x_2)$ is even in x_2. Substituting (3.3) into (3.2) we obtain

$$2a_0 \int_{-\infty}^{\infty} \frac{\sin pb}{p^2} \operatorname{cth} p\gamma\pi\, dp + \sum_{m=1}^{\infty} a_m \left[f_m(x_2) + p_m \cos \frac{m\pi}{b}(x_2 + b) \right] - \varphi(x_1, x_2) - \frac{\bar{q}_1(x_1)}{2h} \int_{-\infty}^{\infty} \frac{\cos px_2}{p^2}\, dp = 0 \tag{3.4}$$

$$f_m(x_2) = -2(-1)^m \int_{-\infty}^{\infty} \operatorname{cth} p\gamma\pi \frac{\sin pb}{\delta_m^2 - p^2} \cos px_2\, dp$$

$$p_m = 2b \frac{(-1)^m}{m} \operatorname{cth} 2\alpha m\pi; \qquad \delta_m = \frac{m\pi}{b}$$

In order to calculate $a_0, a_1, \ldots$, we shall require that the right-hand side of the relation (3.4) be orthogonal, on the interval $|x_2| \leqslant b$, to the complete system of functions $\cos(n\pi x_2 b^{-1})$ $(n = 0, 1, 2 \ldots)$ /5/. Multiplying the left and right part of (3.4) by each function and integrating over the limits shown, we obtain

$$\sum_{m=0}^{\infty} g_{0m} a_m = D_0, \quad \sum_{m=0}^{\infty} g_{nm} a_m + \frac{4b^2}{n^2} \operatorname{cth}(2\alpha n\pi)\, a_n = D_n \tag{3.5}$$

$$g_{00} = 4 \int_{-\infty}^{\infty} \chi_0(p)\, dp, \qquad \chi_0(p) = \frac{\sin^2 pb}{p^3} \operatorname{cth} p\gamma\pi$$

$$g_{0m} = g_{m0} = -4(-1)^m \int_{-\infty}^{\infty} \frac{\sin^2 pb}{p(\delta_m^2 - p^2)} \operatorname{cth} p\gamma\pi\, dp$$

$$g_{nm} = 4(-1)^{m+n} \int_{-\infty}^{\infty} \frac{p \sin^2 pb}{(\delta_m^2 - p^2)(\delta_n^2 - p^2)} \operatorname{cth} p\gamma\pi\, dp \qquad (m = 1, 2, \ldots)$$

$$D_0 = \int_{-b}^{b} \varphi(x_2)\,dx_2 + \frac{\bar{q}_1(x_1)}{h}\int_{-\infty}^{\infty} \chi_1(p)\,dp, \qquad \chi_1(p) = \frac{\sin pb}{p^3}$$

$$D_n = \int_{-b}^{b} \varphi(x_2)\cos\frac{n\pi x_2}{b}\,dx_2 \qquad (n = 1, 2, \ldots)$$

The system (3.5) representes an infinite system of equations for $a_0, a_1, \ldots$. The functions $\chi_0(p)$ and $\chi_1(p)$ appearing under the integral sign in the formulas for g_{00} and D_0 have second order power singularities when $p \to 0$. From this it follows that the system (3.5) has a solution, if

$$\lim_{p\to 0}\left[a_0\chi_0(p) - \frac{q_1(x_1)\chi_1(p)}{h}\right] = 0$$

and hence $a_0 = \bar{q}_1(x_1)/(2b)$. The last result fully agrees with the second relation of (1.4) and underlines the absence of contradictions in the approach used. It should be noted that the left-hand part of the system (3.5) does not depend on the type of the external load. Moreover, as the numerical analysis indicates, the system (3.5) is regular /5/ and can therefore be easily solved by truncation method.

4. We consider, as an example, a reinforced half-layer (Fig.1) with the following boundary conditions given at its end face:

$$\sigma_{11} = Ef(x_3)\cos\frac{x_2}{l}; \quad u_2 = u_3 = 0 \quad \text{for } x_1 = 0 \tag{4.1}$$

In this case the averaged boundary conditions for (1.6) have the form

$$x_1 = 0, \quad \sigma_{11*} = EQ\cos\frac{x_2}{l}, \quad u_{2*} = 0 \quad \left(Q = \frac{1}{2h}\int_{-h}^{h} f(x_3)\,dx_3\right) \tag{4.2}$$

Taking into account the fact that the load (4.2) is symmetric in x_2, we can formulate the boundary value problem (1.6), (4.2) as a plane problem for a plane quadrant, writing the boundary conditions at $x_2 = 0$ in the form of conditions for a reinforced edge

$$L_{j*}(u_{1*}^n, u_{2*}^n) = 0; \quad \sigma_{12*}^n + \frac{EF}{4h}\frac{\partial^2 u_{1*}^n}{\partial x_1^2}\bigg|_{x_2=0} = 0, \quad u_{2*}^n\big|_{x_2=0} = 0 \tag{4.3}$$

To solve the boundary value problem (4.2), (4.3), we apply infinite sine and cosine Fourier transforms in x_1 to the equations and boundary conditions (4.3). As a result, we obtain the following system of ordinary differential equations in terms of the transforms

$$\bar{u}_1(p, x_2) = \int_0^\infty u_{1*}^n(x_1, x_2)\cos p\xi_1\,dp$$

$$\bar{u}_2(p, x_2) = \int_0^\infty u_{2*}^n(x_1, x_2)\sin p\xi_1\,dp \qquad (\xi_j = x_j/l)$$

with the corresponding boundary conditions. Solving this system and returning to the original variables, we obtain

$$u_{1*}^n = -\frac{2}{\pi}\int_0^\infty T(p)\left(\xi_2 - \frac{\lambda}{p}\right)\exp(-p\xi_2)\cos p\xi_1\,dp - \frac{\beta_1}{2}\,\frac{(1+\nu)\xi_1 + 3 + \nu}{1-\nu}\exp(-\xi_1)\cos\xi_2 \tag{4.4}$$

$$u_{2*}^n = \frac{2}{\pi}\int_0^\infty T(p)\,\xi_2\exp(-p\xi_2)\sin p\xi_1\,dp - \frac{\beta_1}{2}\,\frac{1+\nu}{1-\nu}\,\xi_1\exp(-\xi_1)\sin\xi_2$$

$$T(p) = \frac{\alpha_1\beta_1 p^2[(1-\nu)p^2 + 2]}{(1-\nu)(1+\lambda+\alpha_1\lambda p)(1+p^2)^2}, \quad \alpha_1 = \frac{EF}{4\mu lh}$$

$$\beta_1 = (1-\nu^2)Ql, \quad \lambda = \frac{3-\nu}{1-\nu}$$

The averaged contact force of interaction between the stringer and the plate is

$$\bar{q}_1(x_1) = EF\frac{\partial^2 u^n_{1*}(x_1, 0)}{\partial x_1^2} = \frac{2EF\beta_1(1+\lambda)}{\alpha_1 \pi l^2}\int_0^\infty T(p)\cos p\xi_1\,dp \qquad (4.5)$$

Substituting the averaged values obtained into the right-hand side of the system (3.5) and solving it with help of the method indicated, yields the coefficients $a_1, a_2, \ldots$.

The results obtained were computed numerically for the following initial values of the parameters:

$$d = 1,\quad b = h = 0.5,\quad l = 10,\quad E = 10^6,\quad \nu = 0.3,\quad Q = 10^{-2}$$

The number of equations in the system (3.5) was varied, with the results however remaining stable and practically unchanged by increasing the number of equations. The values of the first coefficients of the expansion (3.3) were as follows:

$$a_0 = \bar{q}_1 = 504,\ a_1 = 6.99,\quad a_2 = 56.9,\ a_3 = -31.9,\ a_4 = 22.6,\ a_5 = -17.7,\ a_6 = 14.5,\ a_7 = -12.4,\ a_8 = 10.8,\quad a_9 = -9.54.$$

Formulas (4.4) together with the relations of the Hooke's Law for the generalized plane stress state, yield analytic expressions for the averaged stresses $\sigma_{11*}, \sigma_{12*}, \sigma_{22*}$. From the asymptotic analysis it follows that the basic stresses of the additional state are $\sigma_{12}{}^{ng}, \sigma_{13}{}^{ng}, \sigma_{12}{}^{pg}, \sigma_{13}{}^{pg}$. They are expressed in terms of $V_1^{(2)}, W_1^{(2)}$ by the formulas

$$\sigma_{12}^{ng} \approx \mu\frac{\partial V_{10}}{\partial x_2},\quad \sigma_{13}^{ng} \approx \mu\frac{\partial V_{10}}{\partial x_3},\quad \sigma_{12}^{pg} \approx \mu\frac{\partial W_{10}}{\partial x_2},\quad \sigma_{13}^{pg} \approx \mu\frac{\partial W_{10}}{\partial x_3}$$

Figs.2 and 3 show the results of numerical computations for the stresses σ_{12} and σ_{13} for $x_1 = l/2$. The dashed line corresponds to the averaged stress $\sigma_{12}{}^*$ and the solid line to the additional stresses. Curve *1* is constructed for $x_3 = 0$, *2* for $x_3 = 0.3$, *3* for $x_3 = 0.5$, and *4* for $x_3 = 1$. The shearing stress $\sigma_{13}{}^{ng}|_{x_3=0.5} = \sigma_{13}{}^{pg}|_{x_3=0.5}$ (curve *3*, $|x_2| \leqslant b$) coincides with the contact stress $q_1(l/2, x_2)$. For the purposes of comparison we note that the extremal value (for $x_1 = l/2$) has the averaged stress σ_{11*} which is equal, for $x_2 = 1.2$ to $6.6 \cdot 10^3$.

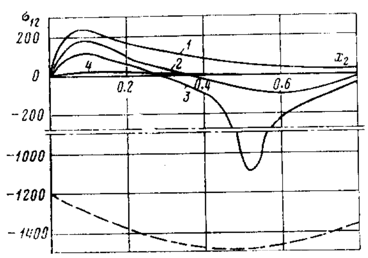

Fig.2

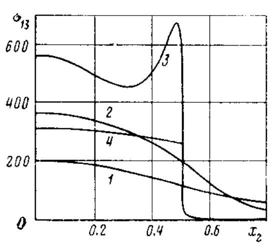

Fig.3

The results obtained imply that the fundamental stresses of the additional state are localized near the zone of contact and are comparable with the shear strength of the averaged state, which can be determined with help of the usual numerical schemes.

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